

# LEPTON FLAVOR VIOLATION IN SUPERSYMMETRIC MODELS WITH NON-DEGENERATE $A$ -TERMS

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We analyze the lepton-flavor violation in supersymmetric models with non-universal soft breaking terms derived from strings. We show that the non-universality of the scalar masses enhances the branching ratios more significantly than the non-universality of the  $A$ -terms.

## 1 Introduction

Non-universal  $A$ -terms provide an interesting framework to enhance the supersymmetric contributions to CP violation effects<sup>1,2,3,4</sup>. It was shown that the flavor structure of the  $A$ -terms is crucial for enhancing the SUSY contributions to CP violation effects, and for generating the experimentally observed  $\varepsilon$  and  $\varepsilon'/\varepsilon$ . These models also predict a CP asymmetry in  $B \rightarrow X_s \gamma$  decay much larger than the one predicted by SM and the one obtained for a wide region of the parameter space in minimal supergravity scenarios<sup>5</sup>. It has also been noted that the analysis of  $b \rightarrow s \gamma$  does not severely constrain the models under consideration<sup>6</sup>. However, the non-universality of the  $A$ -terms in this class of models is always associated with non-universal scalar masses. In this case a simple non-diagonal Yukawa texture predicting lepton masses will induce two sources of LFV: one due to the flavor structure of the  $A_l$ -terms which prohibits the simultaneous diagonalization of the lepton Yukawa matrices  $Y_l$  and the trilinear couplings  $(Y_l^A)_{ij} \equiv (A_l)_{ij}(Y_l)_{ij}$ ; the other source is due to the non degeneracy of the scalar masses of the sleptons. Therefore, in the basis where  $m_l$  is diagonal the slepton mass matrix acquires non-diagonal contributions. We find that in general the second source dominates over the first in the case of the LFV predictions.

## 2 String inspired models with non-degenerate $A$ -terms

In this work we consider the class of string inspired model which has been recently studied in Refs.<sup>1,2,3</sup>. In this class of models, the trilinear  $A$ -terms of the soft SUSY breaking are non-universal. It was shown that this non-universality among the  $A$ -terms plays an important role on CP violating processes. In particular, it has been shown that non-degenerate  $A$ -parameters

can generate the experimentally observed CP violation  $\varepsilon$  and  $\varepsilon'/\varepsilon$  even with a vanishing  $\delta_{\text{CKM}}$ .

Here we consider two models for non-degenerate  $A$ -terms. The first model (model A) is based on weakly coupled heterotic strings, where the dilaton and the moduli fields contribute to SUSY breaking <sup>7</sup>. The second model (model B) is based on type I string theory where the gauge group  $SU(3) \times U(1)_Y$  is originated from the 9 brane and the gauge group  $SU(2)$  is originated from one of the 5 branes <sup>8</sup>.

### 2.1 Model A

We start with the weakly coupled string-inspired supergravity theory. In this class of models, it is assumed that the superpotential of the dilaton ( $S$ ) and moduli ( $T$ ) fields is generated by some non-perturbative mechanism and the  $F$ -terms of  $S$  and  $T$  contribute to the SUSY breaking. Then one can parametrize the  $F$ -terms as <sup>7</sup>

$$F^S = \sqrt{3}m_{3/2}(S + S^*) \sin \theta, \quad F^T = m_{3/2}(T + T^*) \cos \theta. \quad (1)$$

Here  $m_{3/2}$  is the gravitino mass,  $n_i$  is the modular weight and  $\tan \theta$  corresponds to the ratio between the  $F$ -terms of  $S$  and  $T$ . In this framework, the soft scalar masses  $m_i$  and the gaugino masses  $M_a$  are given by <sup>7</sup>

$$m_i^2 = m_{3/2}^2(1 + n_i \cos^2 \theta), \quad (2)$$

$$M_a = \sqrt{3}m_{3/2} \sin \theta. \quad (3)$$

The  $A^{u,d}$ -terms are written as

$$(A^{u,d})_{ij} = -\sqrt{3}m_{3/2} \sin \theta - m_{3/2} \cos \theta(3 + n_i + n_j + n_{H_{u,d}}), \quad (4)$$

where  $n_{i,j,k}$  are the modular weights of the fields that are coupled by this  $A$ -term. If we assign  $n_i = -1$  for the third family and  $n_i = -2$  for the first and second families (we also assume that  $n_{H_1} = -1$  and  $n_{H_2} = -2$ ) we find the following texture for the  $A$ -parameter matrix at the string scale

$$A^{u,d} = \begin{pmatrix} x_{u,d} & x_{u,d} & y_{u,d} \\ x_{u,d} & x_{u,d} & y_{u,d} \\ y_{u,d} & y_{u,d} & z_{u,d} \end{pmatrix}, \quad (5)$$

where

$$x_u = m_{3/2}(-\sqrt{3} \sin \theta + 3 \cos \theta), \quad (6)$$

$$x_d = y_u = m_{3/2}(-\sqrt{3} \sin \theta + 2 \cos \theta), \quad (7)$$

$$y_d = z_u = m_{3/2}(-\sqrt{3} \sin \theta + \cos \theta), \quad (8)$$

$$z_d = -\sqrt{3}m_{3/2} \sin \theta. \quad (9)$$

The non-universality of this model is parameterized by the angle  $\theta$  and the value  $\theta = \pi/2$  corresponds to the universal limit for the soft terms. In order

to avoid negative mass squared in the scalar masses we restrict ourselves to the case with  $\cos^2 \theta < 1/2$ . Such restriction on  $\theta$  makes the non-universality in the whole soft SUSY breaking terms very limited. However, as shown in <sup>1,2</sup>, this small range of variation for the non-universality is enough to generate sizeable SUSY CP violations in K system.

## 2.2 Model B

This model is based on type I string theory and like model A, it is a good candidate for generating sizeable SUSY CP violations. In type I string theory, non-universality in the scalar masses,  $A$ -terms and gaugino masses can be naturally obtained <sup>8</sup>. Type I models contain either 9 branes and three types of  $5_i (i = 1, 2, 3)$  branes or  $7_i$  branes and 3 branes. From the phenomenological point of view there is no difference between these two scenarios. Here we consider the same model used in Ref. <sup>3</sup>, where the gauge group  $SU(3)_C \times U(1)_Y$  is associated with 9 brane while  $SU(2)_L$  is associated with  $5_1$  brane.

If SUSY breaking is analysed, as in model A, in terms of the vevs of the dilaton and moduli fields <sup>8</sup>

$$F^S = \sqrt{3}m_{3/2}(S + S^*)\sin\theta, \quad F^{T_i} = m_{3/2}(T_i + T_i^*)\Theta_i \cos\theta, \quad (10)$$

where the angle  $\theta$  and the parameter  $\Theta_i$  with  $\sum_i |\Theta_i|^2 = 1$ , just parametrize the direction of the goldstino in the  $S$  and  $T_i$  fields space. Within this framework, the gaugino masses are <sup>8</sup>

$$M_1 = M_3 = \sqrt{3}m_{3/2}\sin\theta, \quad (11)$$

$$M_2 = \sqrt{3}m_{3/2}\Theta_1 \cos\theta. \quad (12)$$

In this case the quark doublets and the Higgs fields are assigned to the open string which spans between the  $5_1$  and 9 branes. While the quark singlets correspond to the open string which starts and ends on the 9 brane, such open string includes three sectors which correspond to the three complex compact dimensions. If we assign the quark singlets to different sectors we obtain non-universal  $A$ -terms. It turns out that in this model the trilinear couplings  $A^u$  and  $A^d$  are given by <sup>8,3</sup>

$$A^u = A^d = \begin{pmatrix} x & y & z \\ x & y & z \\ x & y & z \end{pmatrix}, \quad (13)$$

where

$$x = -\sqrt{3}m_{3/2}(\sin\theta + (\Theta_1 - \Theta_3)\cos\theta), \quad (14)$$

$$y = -\sqrt{3}m_{3/2}(\sin\theta + (\Theta_1 - \Theta_2)\cos\theta), \quad (15)$$

$$z = -\sqrt{3}m_{3/2}\sin\theta. \quad (16)$$

The soft scalar masses for quark-doublets and Higgs fields ( $m_L^2$ ), and the quark-singlets ( $m_{R_i}^2$ ) are given by

$$m_L^2 = m_{3/2}^2 \left( 1 - \frac{3}{2}(1 - \Theta_1^2) \cos^2 \theta \right), \quad (17)$$

$$m_{R_i}^2 = m_{3/2}^2 (1 - 3\Theta_i^2 \cos^2 \theta), \quad (18)$$

where  $i$  refers to the three families. For  $\Theta_i = 1/\sqrt{3}$  the  $A$ -terms and the scalar masses are universal while the gaugino masses could be non-universal. The universal gaugino masses are obtained at  $\theta = \pi/6$ .

In models with non-degenerate  $A$ -terms we have to fix the Yukawa matrices to completely specify the model. Here we assume that the Yukawa texture has the following form

- Texture I,

$$Y_l = y^\tau \begin{pmatrix} 0 & 5.07 \times 10^{-3} & 0 \\ 5.07 \times 10^{-3} & 8.37 \times 10^{-2} & 0.4 \\ 0 & 0.4 & 1 \end{pmatrix} \quad (19)$$

- Texture II,

$$Y_l = y^\tau \begin{pmatrix} 3.3 \times 10^{-4} & 1.64 \times 10^{-5} & 0 \\ 1.64 \times 10^{-5} & 8.55 \times 10^{-2} & 0.4 \\ 0 & 0.4 & 1 \end{pmatrix}. \quad (20)$$

Both of them lead to the correct prediction for the experimental values of the lepton masses.

### 3 Lepton flavor violation vs. non-universality

We are now ready to investigate the lepton flavor violation processes  $\tau \rightarrow \mu \gamma$  and  $\mu \rightarrow e \gamma$ <sup>9</sup>. Despite lepton flavor is preserved in the standard model (SM), SUSY models can predict branching ratios for these processes compatible with present experimental bounds as it has been discussed by J. Casas in this conference. The details of the calculations for these ratios in the context of the models described in section 2 are in Ref.<sup>10</sup>. As emphasized in section 2, the splitting of the soft masses in model A increases from  $\sin \theta = 1$  (which corresponds to the universal case) to the limiting case for  $\sin \theta = 1/\sqrt{2}$  (below which some square masses to become negative). Therefore we consider as representative for this model to present the variation of the branching ratios with  $\sin \theta$  for fixed values of  $m_{3/2}$  as shown in Fig. 1.

For the value  $m_{3/2} = 200$  GeV, the mass of lightest neutralino varies from 100 to 147 GeV, that of the chargino from 190 to 277 GeV, while the lightest of the staus has masses of 107 to 233 GeV as  $\sin \theta$  ranges from  $1/\sqrt{2}$  to 1. Similarly for  $m_{3/2} = 400$  GeV we found  $m_{\tilde{\chi}^0} = 210 - 295$  GeV,  $m_{\tilde{\chi}^+} = 400 - 570$  GeV,  $m_{\tilde{\tau}_2} = 212 - 470$  GeV for the same range of  $\sin \theta$ .

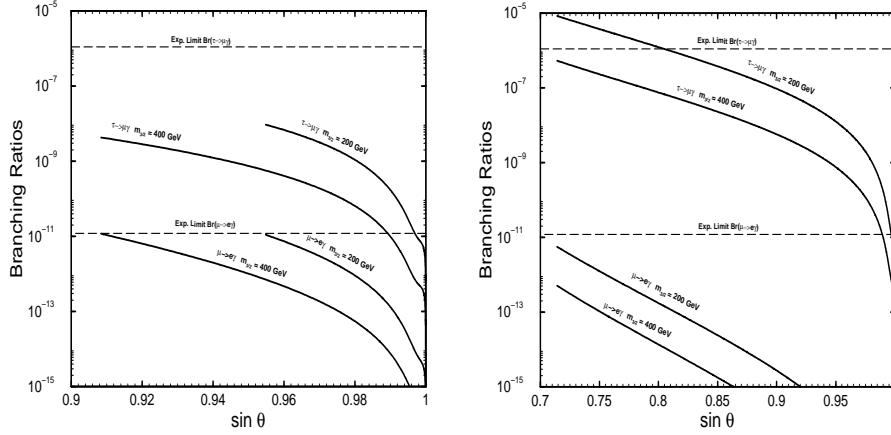


Figure 1. Branching ratios vs.  $\sin \theta$  for model A with texture I for  $Y_l$  (left) and texture II (right) and  $\tan \beta = 10$ . The values for  $m_{3/2}$  are kept constant as shown on the curves.

From this figure we can see how texture I (graphic on the left) allows small deviations from universality of the soft terms. The experimental bound on  $BR(\mu \rightarrow e\gamma)$  is satisfied only for  $\sin \theta > .96$  ( $m_{3/2} = 200$  GeV) and for  $\sin \theta > .91$  ( $m_{3/2} = 400$  GeV) while for the same range on  $\sin \theta$  the corresponding prediction for  $BR(\tau \rightarrow \mu\gamma)$  is well below the experimental bound. The values of the branching ratios decrease as we increase  $m_{3/2}$  and  $\sin \theta$  since this translates into an increase of the masses of the supersymmetric particles. In order to simplify the presentation of our results we fix  $\tan \beta = 10$ . However, enlarging the value of  $\tan \beta$  increases the prediction for the branching ratios.

The results obtained using texture II (Fig. 2, graphic on the right) allow us to start the graph at the lowest value of  $\sin \theta = 1/\sqrt{2}$ . As it can be seen, the experimental bounds are more restrictive for the  $\tau \rightarrow \mu\gamma$  than for  $\mu \rightarrow e\gamma$  process.

We turn now to study the lepton flavor violation in model B. In this model, the structure of the soft-terms is more complicated than in the previous model. They depend, in addition to  $m_{3/2}$  and  $\theta$ , on the values of the parameters  $\Theta_1$ ,  $\Theta_2$  and  $\Theta_3$ . However, the flavor structure of the slepton matrices is simpler, since the soft masses for the left-handed sleptons are universal at the GUT scale and the sneutrino mass matrix remains diagonal under a rotation that diagonalizes  $Y_l$ . Therefore diagram with exchanging sneutrino will not contribute to the LFV processes. Also we found that a bound of  $m_{\tilde{\chi}^+} = 95$  GeV is found to be the most severe on the initial conditions. The predictions we obtain with this model for  $BR(l_j \rightarrow l_i\gamma)$  allow us to simplify

our presentation by setting  $\Theta \equiv \Theta_1 = \Theta_2$ . In the case of texture I, this is justified by the fact that the experimental bound on  $BR(\mu \rightarrow e\gamma)$  tolerates a small deviation of the  $\Theta_i$  's from the common value of  $1/\sqrt{3}$ , for which the soft masses become universal as shown in Fig. 2.

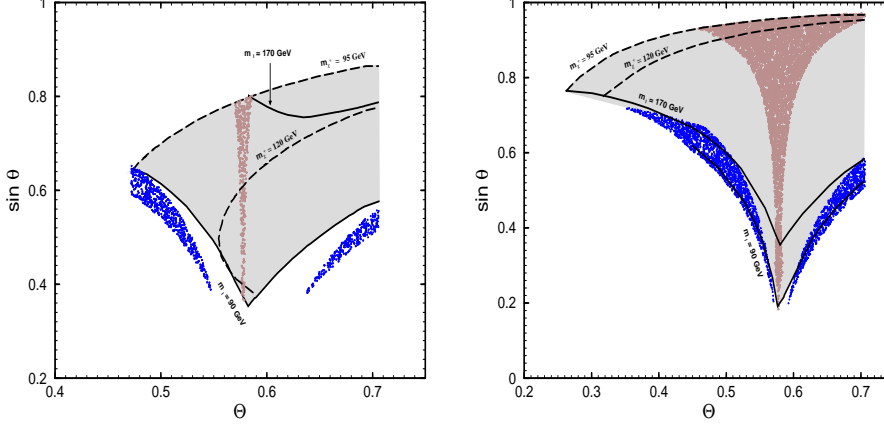


Figure 2. Areas with  $BR(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11}$  (dotted areas inside the gray contour) in the plane  $\sin \theta - \Theta$  for constant values of  $m_{3/2} = 200$  GeV (left) and  $m_{3/2} = 400$  GeV (right) and  $\tan \beta = 10$ . The model used corresponds to type I string, with texture I for  $Y_i$ . Values of the masses of SUSY particles which bound the parameter space of the model are as shown in the graphs. The dark dotted areas correspond to space of parameters such that the LSP is a slepton.

For the case of texture II, these predictions are more tolerant to a variation of the  $\Theta_i$  's. However, when this texture is considered, the experimental limit on  $BR(\tau \rightarrow \mu\gamma)$  is more restrictive since this bound is particularly sensitive to the value  $\Theta_3$ . Therefore, we find that setting also  $\Theta_1 = \Theta_2$  in the analysis of our results with texture II, we can achieve a clearer presentation without any loss of generality.

Fig. 2 shows the constraint imposed by the current bound on the  $BR(\mu \rightarrow e\gamma)$  on the plane  $(\sin \theta - \Theta)$  for constant values of  $m_{3/2} = 200$  GeV (left) and  $m_{3/2} = 400$  GeV (right) when texture I is assumed. Fig.3 displays the equivalent for texture II. The light shaded areas correspond to the space of parameters allowed by the bounds on the masses of the SUSY particles. The region below the upper dashed line corresponds to values of  $m_{\tilde{\chi}^+} > 95$  GeV, while the sector above the lower solid line corresponds to values of the lightest charged scalar  $m_{\tilde{l}} > 90$  GeV.

The shape of the curve  $m_{\tilde{l}} = 90$  GeV in Figs. 2 and 3 is determined by the initial conditions given above. The lowest values for these masses corresponds to  $m_{R_1} = m_{R_2}$  when  $1/\sqrt{3} < \Theta < 1/\sqrt{2}$ , while for  $\Theta < 1/\sqrt{3}$ ,  $m_{R_3}$  is the lowest value. Therefore the largest component of the lowest eigenvalue of

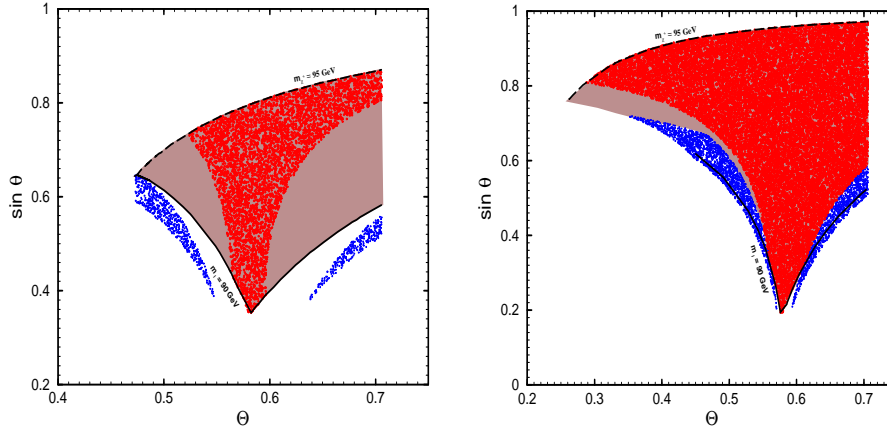


Figure 3. Areas with  $BR(\tau \rightarrow \mu\gamma) < 1.1 \times 10^{-6}$  (dotted areas inside the gray contour) in the plane  $\sin \theta - \Theta$  for constant values of  $m_{3/2} = 200$  GeV (left) and  $m_{3/2} = 400$  GeV (right) and  $\tan \beta = 10$ . The model used corresponds to type I string, with texture II for  $Y_l$ . Values of the masses of SUSY particles which bound the parameter space of the model are as shown in the graphs. The dark dotted areas correspond to space of parameters such that the LSP is a slepton.

the charged slepton mass is the  $\tilde{e}_R$  or the  $\tilde{\tau}_R$  depending on the ranges of  $\Theta$  above. Similar considerations explain the different shape of the curves for  $m_{\tilde{l}} = 170$  GeV (with  $m_{3/2} = 200$  GeV and  $m_{3/2} = 400$  GeV).

The darkest dotted areas in Figs. 2 and 3 represent the sector of parameters for which the lightest supersymmetric particle (LSP) is a charged slepton. For  $m_{3/2} = 200$  GeV these areas are below the bound of  $m_{\tilde{l}} = 90$  GeV. However for  $m_{3/2} = 400$  GeV, the cosmological requirement on the LSP to be a neutral particle (lightest neutralino in our case) imposes a further restriction on the space of parameters of the model.

Similarly to the results found for the model A, the assumption of texture I for  $Y_l$  allows a small deviation from the universality of the scalar masses once we impose the experimental bound on  $BR(\mu \rightarrow e\gamma)$  (light dotted sector inside of the grey area in Fig. 2). However we found that the corresponding limit on  $\tau \rightarrow \mu\gamma$  does not constraint the space of parameters shown in Fig. 3. The fact that the branching ratios decrease with  $m_{3/2}$  is reflected in a wider light dotted area on the graphic corresponding to  $m_{3/2} = 400$  GeV in Fig. 2.

#### 4 Conclusions

We have studied the predictions for the LFV decays  $\mu \rightarrow e\gamma$  and  $\tau \rightarrow \mu\gamma$  arising from non universal soft terms. We also showed the dependence of these predictions on a general, non-diagonal, texture of the lepton yukawa

couplings.

We found the non-universality of the soft masses is more relevant for LFV than those of the  $A$ -term are. However, the latter ones are of phenomenological interest for other processes such as CP violation effects.

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